QUINTESSENCE DARK ENERGY INSPIRED BY DUAL ROLES OF THE RICCI SCALAR

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Abstract

In this letter, dark energy is obtained using dual roles of the Ricci scalar (physical as well as geometrical role of the Ricci scalar curvature) in a natural way without any input in the theory for it. In contrast to other models, where an idea for dark energy is introduced in the beginning of the theory, here it emerges from the gravitational sector spontaneously. Dark energy density, obtained in this model, decreases with the scale factor $a(t)$ as $\sim a(t)^{-3(1+\mathrm{w}_{(\mathrm{de})})}$ (where $\mathrm{w}_{(\mathrm{de})} > -1$ is the equation of state parameter for dark energy) explaining fall of its value from $1.19 \times 10^{75}\mathrm{GeV}^4$ at Planck scale to its present value suggested by astronomical observations. Interestingly, apart from dark energy, dark radiation and dark matter density are also obtained from the gravity sector. Moreover, time for transition from deceleration to acceleration of the universe is evaluated. It is found that if dark energy obeys equation of state for generalized Chaplygin gas and barotropic fluid simultaneously, after tansition time, the scale factor will evolve as $cosh(t - t_*)$(t and t_* being time and transition time respectively),

but after a finite time expansion will become de-Siiter like. PACS no. 98.80.Cq

1. Astronomical observations, during the last few years, provide compelling evidences in favor of accelerating universe at present, which is caused by dominance of dark energy (DE) [1, 2, 3, 4, 5]. Observation of 16 Type Ia supernovae (SNe Ia) by *Hubble Space Telescope* further modifies these results and shows evidence for cosmic deceleration preceding acceleration in the late universe [6]. So, DE has a significant role in cosmic dynamics of the current universe. The simplest candidate for DE is supposed to be the cosmological constant Λ which is very high in the early universe, but there is no mechanism to bring it to present value without fine-tuning. Alternatively, to explain its decay from a very high value, in the early universe, to its present extremely small value, many models [7, 8] were suggested in which Λ is envisaged as a slowly varying function of cosmic time. Apart from dynamical Λ as DE, other models are fluid-dynamics models, where barotropic fluid is its source with or without dissipative pressure , Chaplygin gas(GC) and generalized Chaplygin gas (GCG)models [9, 10, 11, 12]. In field-theoretic models, the most natural ones are models, where DE is caused by scalars. These are quintessence models [13], k-essence models [14], tachyon models [15] and phantom models [12, 16]. In these models, non-gravitational lagrangian density for exotic matter giving DE is added to Einstein-Hilbert term in the action. Recently, in a different appraoch, non-gravitational term is replaced by gravitational term being non-linear in Ricci scalar R, which stems modified gravity [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28].

In all these models, a non-gravitational or gravitational term for DE is added in the theory *a priori*. In this sense, these models are phenomenological. Cosmology demands a DE model, where DE density $\rho_{(\rm de)}$ emerges spontaneously from a basic theory. With this motivation, here,$\rho_{(\rm de)}$ is derived from the gravitational sector without an input for it. Here also, a quardatic term of R is added to Einstein-Hilbert term in the action, but not as lagrangian density for DE. It is unlike the approach in [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. In the pesent model, DE emerges spontaneously due to presence of term quardatic in R manifesting dual role of the Ricci scalar as a physical field as well as geometry.

Natural units ($\hbar = c = 1$) are used here with GeV as the fundamental unit, where $\hbar$ and c have their usual meaning. In this unit, it is found that $1{\rm GeV}^{-1} = 6.58 \times 10^{-25}$sec.

2.The action for higher-derivative gravity is taken as

$$S_g = \int d^4x\sqrt{-g}\Big[\frac{R}{16\pi G} - \alpha(x)R^2\Big], \tag{1}$$

where R is the Ricci scalar curvature, $G = M_P^{-2}(M_P = 10^{19}$ GeV is the Planck mass). Moreover, dimensionless α is a scalar depending space-time coordintaes.

The action (1) yields gravitational field equations

$$\frac{1}{16\pi G}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) - \alpha(2\nabla_\mu\nabla_\nu R - 2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu})$$
$$-2(\nabla_\mu\nabla_\nu\alpha - g_{\mu\nu}\Box\alpha)R - 4(\nabla_\mu\alpha\nabla_\nu R - g_{\mu\nu}\nabla^\sigma\alpha\nabla_\sigma R) = 0 \tag{2}$$

using the condition $\delta S_g/\delta g^{\mu\nu} = 0$. Here, ∇_μ denotes covariant derivative and the operator $\Box$ is given as

$$\Box = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\Big(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\Big) \tag{3}$$

with $\mu, \nu = 0, 1, 2, 3$ and $g_{\mu\nu}$ as metric tensor components.

Taking trace of eqs.(2), it is obtained that

$$\Box R + m^2 R + \frac{2}{\alpha} \nabla^{\nu} \alpha \nabla_{\nu} R = 0, \tag{4a}$$

where

$$m^2 = \frac{1}{96\pi G \alpha(t)} + \frac{\Box \alpha}{\alpha} \tag{4b}$$

with $\alpha \neq 0$ to avoid the ghost problem. Here, overdot gives derivative with respect to time t.

Eq.(4a) shows that the Ricci scalar R behaves as a physical field also with (mass)^2 depending on G, in addition to its usual role as a geometrical field [29, 30, 31].

Experimental evidences support spatially homogeneous flat model of the universe [32]. So, the line-element, giving geometry of the universe, is taken as

$$dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \tag{5}$$

with $a(t)$ as the scale factor.

In the space-time, given by eq.(5), eq.(4a) is obtained as

$$\ddot{R} + \Big(3\frac{\dot{a}}{a} + 2\frac{\dot{\alpha}}{\alpha}\Big)\dot{R} - \Big\{\frac{1}{96\pi G\alpha(t)} + \frac{\ddot{\alpha}}{\alpha} + 3\frac{\dot{a}}{a}\frac{\dot{\alpha}}{\alpha}\Big\}R = 0, \tag{6}$$

In most of the situations, for example, radiation model, matter-dominated model, and accelerated models, we have $a(t)$ as a power-law solution yielding R as the power-law function of $a(t)$. So, it is reasonable to take R as

$$R = \frac{A}{R^n} \tag{7}$$

with $n \neq 0$ being a real number. Moreover, α is taken as

$$\alpha = Da^r, \tag{8}$$

where r is a non-zero real number. Advantage for taking α in this form is mentioned below. Here A and D are constants.

R and α, given by eqs.(7) and (8) respectively, satisfy eq.(6), if

$$\frac{\ddot{a}}{a} + (2 + r - n)\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{96\pi GD(n-r)a^r}, \tag{9}$$

which integrates to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{C}{a^{2[3-(n-r)]}} - \frac{1}{48\pi GD(n-r)[(6-r)-2(n-r)]a^r} \tag{10}$$

with C being an integration constant. Eq.(10) gives dynamics of the universe, which is the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{\rm de} + \rho_{r(m)}) \tag{11}$$

with

$$\rho_{\rm de} = \frac{3C}{8\pi G a^{2[3-(n-r)]}} \tag{12a}$$

and

$$\rho_{r(m)} = \frac{3}{8\pi G}\Big[- \frac{1}{48\pi GD(n-r)[(6-r)-2(n-r)]a^r}\Big]. \tag{12b}$$

It is interesting to note that $\alpha(t) \propto a^r$, gives energy density in the known form $\propto a^{-r}$.

$\rho_{r(m)}$, given by eq.(12b), takes the known form of energy density in two cases (i) $r = 4$ and (ii) $r = 3$. When $r = 4$, it takes the form of radiation density $\rho_r \sim a^{-4}$. It is in the form of matter (pressureless fluid) density $\rho_m \sim a^{-3}$, when $r = 3$.

$\rho_{\rm de}$, given by (12a), emerges from the gravitational sector without using a source for it. So,it is recognized as DE density.

Conservation equation for DE

$$\dot{\rho}_{\rm (de)} + 3\frac{\dot{a}}{a}\rho_{\rm (de)}(1 + {\rm w}_{\rm (de)}) = 0, \tag{13}$$

where equation of state (EOS) parameter ${\rm w}_{\rm (de)} = p_{\rm (de)}/\rho_{\rm (de)}$ with $p_{\rm (de)}$ and $\rho_{\rm (de)}$ being isotropic pressure and density for DE respectively.

Eq.(13) yields

$$2[3-(n-r)] = 3(1+\mathrm{w}_{(\mathrm{de})}), \tag{14}$$

Thus, eqs.(12a) and (14) imply

$$\rho_{\mathrm{de}} = \frac{3C}{8\pi G a^{3(1+\mathrm{w}_{(\mathrm{de})})}}. \tag{15}$$

WMAP data give the present value of DE to be $\rho^0_{(\mathrm{de})} = 0.73\rho_{\mathrm{cr.}}$, where $\rho_{\mathrm{cr.}} = 3H_0^2/8\pi G$ with $H_0 = 100h\mathrm{km/secMpc} = 2.33\times 10^{-42}\mathrm{GeV}$ and $h = 0.68$ (having maximum likelihood) [34, 35]. Using these values, eq.(17) is obtained as

$$\rho_{\mathrm{de}} = 0.73\rho_{\mathrm{cr.}}\Big(\frac{a_0}{a}\Big)^{3(1+\mathrm{w}_{(\mathrm{de})})}. \tag{16}$$

At Planck scale, $\rho_{(\mathrm{de})} = \frac{3}{8\pi G}M_P^2$, so eq.(24) yields

$$\Big(\frac{a_0}{a_P}\Big)^{3(1+\mathrm{w}_{(\mathrm{de})})} = \frac{M_P^2 H_0^{-2}}{0.73} = 5.46\times 10^{121}, \tag{17}$$

where a_P is the scale factor at Planck time $t_P = M_P^{-1}$.

2(a). When $r = 4$, eq.(12b) is obtained as

$$\rho_r = \frac{3}{8\pi G}\Big[\frac{1}{72\pi GD(1-\mathrm{w}_{\mathrm{de}})|-1+3\mathrm{w}_{\mathrm{de}}|a^4}\Big], \tag{18}$$

which looks like energy density for radiation emerging from gravity sector. This type of term also arises in brane-gravity inspired Friedmann equation, which is called as 'dark energy density' [33]. So, like brane-gravity, here also ρ_r (given by eq.(18)) is termed as 'dark radiation density'.

Connecting eqs.(11), (12a) and (16), it is obtained that

$$\Big(\frac{\dot a}{a}\Big)^2 = \frac{E}{a^4} + 0.73H_0^2\Big(\frac{a_0}{a}\Big)^{3(1+\mathrm{w}_{(\mathrm{de})})} \tag{19}$$

with $E = 1/\{72\pi GD(1-\mathrm{w}_{\mathrm{de}})|-1+3\mathrm{w}_{\mathrm{de}}|\}$.

For $\rho_r > \rho_{(\mathrm{de})}$, eq.(19) reduces to

$$\Big(\frac{\dot a}{a}\Big)^2 = \frac{E}{a^4}, \tag{20}$$

which integrates to

$$a^2 = a^2(0) + 2t\sqrt{E} \tag{21}$$

with $a(0) = a(t = 0)$. It shows deceleration as $\ddot{a} < 0$.

Investigations start here from the Planck scale (being the fundamental scale).Using $a_P = a(t = t_P)$ with Planck time $t_P = M_P^{-1}$ in eq.(18), C is evaluated as

$$C = \frac{1}{4}(a_P^2 - a^2(0))^2 M_P^2 \simeq \frac{1}{4} a_P^4 M_P^2. \tag{22}$$

as $a(0) < a_P$. Thus,

$$a^2 \simeq a_P^2 M_P t. \tag{23}$$

Connecting eqs.(16), (17), (18) and (22), it is obtained that $\rho_r > \rho_{\rm de}$ if

$$\left(\frac{a_0}{a}\right)^{1-3{\rm w}_{\rm de}} > 5.15 \times 10^{-112} \times (5.46 \times 10^{121})^{4/3(1+{\rm w}_{\rm de})}. \tag{24}$$

For ${\rm w}_{\rm de} = -0.89$, this inequality yields

$$1 + z = \frac{a_0}{a} > 10^{365}, \tag{25}$$

where z is the red-shift.

It is found that universe decelerates when $\rho_r > \rho_{\rm de}$. Moreover, universe accelerates when DE dominates. It means that transition from dceleration to acceleration is possible when transition from $\rho_r > \rho_{\rm de}$ to $\rho_r < \rho_{\rm de}$ takes place. According to (25), it is possible at red-shift 10^{365}, which is extremely large. 16 Type Supernova suggest red-shift for this transition as $z = 0.46 \pm 0.13$ [6]. Thus this case (for $(n-r) = 1$) contradicts experimental results. So, it is ignored here onwards.

2(b). When $r = 3$, eq.(12b) looks like

$$\rho_m = \frac{3B}{8\pi G a^3} \tag{26}$$

with $B = \frac{1}{216\pi G(1-\mathrm{w_{de}})|\mathrm{w_{de}}|}$. The energy density (26) has the form of density for pressueless matter emerging from gravity. So, it is termed as dark matter density (DMD). The prtesent value of DMD is $\rho_m^0 = 0.23\rho_{\rm cr}$ (critical density $\rho_{\rm cr}$ is defined above). So, (26) is obtained as

$$\rho_m = 0.23\rho_{\rm cr}\Big(\frac{a_0}{a}\Big)^3. \tag{27}$$

Now, $\rho_m > \rho_{\rm de}$ when red-shift z is given as

$$z > \Big(\frac{73}{23}\Big)^{1/3|\mathrm{w_{de}}|} - 1. \tag{28}$$

$\rho_m < \rho_{\rm de}$ for

$$z < \Big(\frac{73}{23}\Big)^{1/3|\mathrm{w_{de}}|} - 1. \tag{29}$$

Inequalities (28) and (29) show that transition from $\rho_m > \rho_{\rm de}$ to $\rho_m < \rho_{\rm de}$ at

$$z_* = \Big(\frac{73}{23}\Big)^{1/3|\mathrm{w_{de}}|} - 1 = \Big(\frac{a_0}{a_*}\Big) - 1. \tag{30}$$

Observations yield $-1 < \mathrm{w_{de}} < -0.82$ for quintessence DE. Using these values of $\mathrm{w_{de}}$, (30) yields

$$0.47 < z_* \lesssim 0.599, \tag{31}$$

which is supported by the range $0.33 \lesssim z_* \lesssim 0.59$ given by 16 Type Supernova observations.

Using (16) and (27), Friedmann equation (11) is obtained as

$$\Big(\frac{\dot a}{a}\Big)^2 = H_0^2\Big[0.23\Big(\frac{a_0}{a}\Big)^3 + 0.73\Big(\frac{a_0}{a}\Big)^{3(1+\mathrm{w_{de}})}\Big] \tag{32}$$

using definition of $\rho_{\rm cr}^0$ given above.

When $z > z_*, \rho_m > \rho_{\rm de}$, so (32) reduces to

$$\Big(\frac{\dot a}{a}\Big)^2 = 0.23H_0^2\Big(\frac{a_0}{a}\Big)^3, \tag{33}$$

which integrates to

$$a(t) = a_d\Big[1 + 0.72H_0\Big(\frac{a_0}{a_d}\Big)^{3/2}(t - t_d)\Big]^{2/3} \tag{34}$$

with $a_d = a(t_d)$ being a constant,$a(t)$, given by (34), shows deceleration.

Further, for $z < z_*, \rho_m < \rho_{\rm de}$, so (32) reduces to

$$\Big(\frac{\dot a}{a}\Big)^2 = 0.73H_0^2\Big(\frac{a_0}{a}\Big)^{3(1+{\rm w_{de}})}, \tag{35}$$

which is integrated to

$$a(t) = a_*\Big[1+\frac{3(1+{\rm w_{de}})}{2}H_0\sqrt{0.73}\Big(\frac{a_0}{a_*}\Big)^{3(1+{\rm w_{de}})/2}(t-t_*)\Big]^{2/3(1+{\rm w_{de}})} \tag{36}$$

with $a_* = a(t_*)$. (36) shows acceleration. Here t_* is the time for transition from deceleration to acceleration. Using $a_0 = a(t_0)$ in (36), we obtain that

$$t_0 - t_* = \Big[1 + \frac{3(1+{\rm w_{de}})}{2}H_0\sqrt{0.73}\Big]^{-1}\Big[1 - \Big(\frac{a_*}{a_0}\Big)^{3(1+{\rm w_{de}})/2}\Big]. \tag{37}$$

Connecting (30) and (37), t_* is evaluated as

$$t_* = 0.627t_0 = 8.59{\rm Gyr} \tag{38}$$

for ${\rm w_{de}} = -0.89$ and present age of the universe $t_0 = 13.7$Gyr [34].

3. So far, we have considered DE (obtained here) as a barotropic fluid obeying EOS

$$p_{\rm de} = {\rm w_{de}}\rho_{\rm de}. \tag{39a}$$

Due to negative pressure and being only fluid with supersymmetric generalization , GC and GCG have been strong candidates for DE. But experimental results support GCG [10, 11]. So, GCG model is preferred.

In what follows, consequences of DE fluid (obtained here) are explored if it behaves as GCG [9, 10, 11, 12] and barotropic fluid simultaneously. This type of situation is explored in [12] for non-gravitational

phantom DE. The equation of state (EOS) for GCG is given as

$$p_{\rm de} = -\frac{M^{1+\beta}}{\rho_{\rm de}^{\beta}}, \tag{39b}$$

where $0 < \beta < 1$ for GCG , $\beta = 1$ for CG and M is a constant.

(39a) and (39b) yield

$$\mathrm{w}_{\rm de} = -\Big(\frac{M}{\rho_{\rm de}}\Big)^{1+\beta}. \tag{40}$$

This equation shows dependence of $\mathrm{w}_{\rm de}$ on $\rho_{\rm de}$ varying with $a(t)$. Now, using eq.(16) for $\rho_{\rm de}$ with variable $\mathrm{w}_{\rm de}$, conservation equation (13) looks like

$$3\dot{\mathrm{w}}_{\rm de} + \frac{3}{lna}\Big(\frac{\dot{a}}{a}\Big)(1+\mathrm{w}_{\rm de}) = \frac{3}{lna}\Big(\frac{\dot{a}}{a}\Big)\Big[1 - \Big(\frac{M}{B}\Big)^{1+\beta} e^{3(1+\beta)(1+\mathrm{w}_{\rm de})lna}\Big] \tag{41a}$$

using eq.(40) for $\mathrm{w}_{\rm de}$. Here,

$$B = 0.73\rho_{\rm cr} a_0^{3(1+\mathrm{w}_{\rm de})}. \tag{41b}$$

Eq.(41a) integrates to

$$a^{-3(1+\beta)(1+\mathrm{w}_{\rm de})} = \frac{\tilde{C}}{a^{3(1+\beta)}} + \Big(\frac{M}{B}\Big)^{1+\beta}, \tag{42}$$

where $\tilde{C}$ is an integration constant.

So, eqs.(16), (41b) and (42) yield

$$\rho_{\rm de} = B\Big[\frac{\tilde{C}}{a^{3(1+\beta)}} + \Big(\frac{M}{B}\Big)^{1+\beta}\Big]^{1/(1+\beta)}. \tag{43}$$

Connecting eqs.(40) and (43), it is obtained that

$$\mathrm{w}_{\rm de} = -\Big(\frac{M}{B}\Big)^{1+\beta}\Big[\frac{\tilde{C}}{a^{3(1+\beta)}} + \Big(\frac{M}{B}\Big)^{1+\beta}\Big]^{-1}. \tag{44}$$

At $a = a_0, \mathrm{w}_{\rm de} = \mathrm{w}^0_{\rm de}$, so eq.(44) yields

$$-\Big(\frac{M}{B}\Big)^{1+\beta} = \Big[\Big(\frac{\mathrm{w}^0_{\rm de}}{1+\mathrm{w}^0_{\rm de}}\Big)\frac{\tilde{C}}{a_0^{3(1+\beta)}} - 1\Big]^{-1}. \tag{45a}$$

Connecting eqs.(44) and (45a), we obtain

$$\mathrm{w}_{\rm de} = \Big[\Big(\frac{1+\mathrm{w}^0_{\rm de}}{\mathrm{w}^0_{\rm de}}\Big)\Big(\frac{a_0}{a}\Big)^{3(1+\beta)} - 1\Big]^{-1}. \tag{45b}$$

Eqs.(43) and (45a) yield

$$\rho_{\rm de} = \frac{B\tilde{C}^{1/(1+\beta)}}{a_0^3}\Big[\Big(\frac{a_0}{a}\Big)^{3(1+\beta)} - \Big(\frac{{\rm w}_{\rm de}^0}{1+{\rm w}_{\rm de}^0}\Big)\Big]^{1/(1+\beta)}. \qquad (46a)$$

At $a = a_0, \rho_{\rm de} = \rho^0_{\rm de} = 0.73\rho_{\rm cr}$, so

$$\rho_{\rm de} = 0.73\rho_{\rm cr}\Big[(1+{\rm w}^0_{\rm de})\Big(\frac{a_0}{a}\Big)^{3(1+\beta)} - {\rm w}^0_{\rm de}\Big]^{1/(1+\beta)} \qquad (46b)$$

setting $\tilde{C} = a^{-3{\rm w}^0_{\rm de}(1+\beta)}$.

The constant M, in (40), is evaluated as

$$M^{1+\beta} = -{\rm w}^0_{\rm de}(\rho^0_{\rm de})^{1+\beta} \qquad (47)$$

using ${\rm w}^0_{\rm de} = {\rm w}_{\rm de}(t_0)$. So, connecting (40), (46b) and (47), it is obtained that

$${\rm w}_{\rm de} = {\rm w}^0_{\rm de}\Big[(1+{\rm w}^0_{\rm de})\Big(\frac{a_0}{a}\Big)^{3(1+\beta)} - {\rm w}^0_{\rm de}\Big]^{-1}. \qquad (48)$$

In the changed circumstances, $\rho_m < \rho_{\rm de}$ when

$$\frac{23}{73}\Big(\frac{a_0}{a}\Big)^3 < \Big[(1+{\rm w}^0_{\rm de})\Big(\frac{a_0}{a}\Big)^{3(1+\beta)} - {\rm w}^0_{\rm de}\Big]^{1/(1+\beta)}. \qquad (49)$$

This inequality is obtained using (27) and (46b). It shows that, for GCG , transition from $\rho_m > \rho_{\rm de}$ to $\rho_m < \rho_{\rm de}$ may take place when

$$\Big(\frac{a_0}{a_*}\Big)^{3(1+\beta)}_{\rm GCG} = -{\rm w}^0_{\rm de}\Big[\Big(\frac{23}{73}\Big)^{1+\beta} - (1+{\rm w}^0_{\rm de})\Big]^{-1}. \qquad (50)$$

Taking ${\rm w}^0_{\rm de} = -0.89$ in (50),

$$z_{*({\rm GCG})} = \Big(\frac{a_0}{a_*}\Big) - 1 = 0.64 \qquad (51)$$

for $\beta = 0.04$. This value of red-shift for transition is very near to upper limit 0.59 of z, given in [6]. So, in GCG case, $\rho_m < \rho_{\rm de}$ when $z < z_* = 0.64$. Now, using (46b) and definition of $\rho_{\rm cr}$, Friedmann

Equation looks like

$$\begin{aligned}\Big(\frac{\dot a}{a}\Big)^2 &= 0.73H_0^2\Big[(1+\mathrm{w}^0_{\mathrm{de}})\Big(\frac{a_0}{a}\Big)^{3(1+\beta)} - \mathrm{w}^0_{\mathrm{de}}\Big]^{1/(1+\beta)} \\ &\simeq 0.73H_0^2|\mathrm{w}^0_{\mathrm{de}}|^{1/(1+\beta)}\Big[\Big\{a^{3(1+\beta)} + \frac{(1+\mathrm{w}^0_{\mathrm{de}})}{2|\mathrm{w}^0_{\mathrm{de}}|(1+\beta)}a_0^{3(1+\beta)}\Big\}^2 \\ &\quad - \frac{(1+2\beta)}{4(1+\beta)^2}\Big(\frac{(1+\mathrm{w}^0_{\mathrm{de}})}{|\mathrm{w}^0_{\mathrm{de}}|}\Big)^2 a_0^{6(1+\beta)}\Big],\end{aligned} \tag{52}$$

which is integrated to

$$a(t) = a_{*(\mathrm{GCG})}\Big[\frac{\sqrt{1+2\beta}cosh\theta - 1}{\sqrt{1+2\beta}-1}\Big]^{1/3(1+\beta)}, \tag{53a}$$

where

$$\theta = 3\sqrt{0.73}H_0(1+\beta)|\mathrm{w}^0_{\mathrm{de}}|^{1/2(1+\beta)}(t - t_{*(\mathrm{GCG})}). \tag{53b}$$

Using $\beta = 0.04$ and $z_* = 0.64, t_{*(\mathrm{GCG})}$ is evaluated as

$$t_{*(\mathrm{GCG})} \simeq 0.8t_0 = 10.9\mathrm{Gyr}. \tag{54}$$

Connecting (53a) and (53b), it is obtained that

$$\frac{\dot a}{a} = \sqrt{0.73}H_0|\mathrm{w}^0_{\mathrm{de}}|^{1/2(1+\beta)}\Big[\frac{\sqrt{1+2\beta}tanh\theta}{\sqrt{1+2\beta} - sech\theta}\Big]. \tag{55a}$$

It shows that

$$\frac{\dot a}{a} \simeq \sqrt{0.73}H_0|\mathrm{w}^0_{\mathrm{de}}|^{1/2(1+\beta)} \tag{55b}$$

for $\theta = 12.2$. Using θ, from (53b), The corresponding time t_{ds}, for $\theta = 12.2$, is obtained as

$$t_{\mathrm{ds}} = t_{*(\mathrm{GCG})} + \frac{4.76H_0^{-1}}{(1+\beta)}|\mathrm{w}^0_{\mathrm{de}}|^{-1/2(1+\beta)}. \tag{56}$$

Using $\beta = 0.04$ and $\mathrm{w}^0_{\mathrm{de}} = -0.89$ in (50), t_{ds} is evaluated as

$$t_{\mathrm{ds}} = t_{*(\mathrm{GCG})} + 4.88t_0 \simeq 5.68t_0 = 77.78\mathrm{Gyr}, \tag{57}$$

where $t_{*(\mathrm{GCG})}$ is given by (54).

For $t \geq t_{\rm ds}, tanh\theta = 1$ and $cosh\theta \simeq \frac{e^{\theta}}{2}$, so $a(t)$ (given by (53a)) looks like

$$a(t) = a_{*({\rm GCG})}\Big[\frac{\sqrt{1+2\beta}cosh\theta - 2}{\sqrt{1+2\beta} - 2}\Big]^{1/3(1+\beta)} \simeq Fe^{\sqrt{0.73}H_0|{\rm w}^0_{\rm de}|^{1/2(1+\beta)}(t - t_{*({\rm GCG})})} \tag{58}$$

where $F = a_{*({\rm GCG})}\Big[\frac{\sqrt{1+2\beta}}{\sqrt{1+2\beta}-2}\Big]^{1/3(1+\beta)}$ with θ, given by (53a). Moreover, for $t \geq t_{\rm ds}$,

$$\rho_{\rm de(GCG)} \simeq 0.73\rho_{\rm cr}|{\rm w}^0_{\rm de}|^{1/(1+\beta)}. \tag{59}$$

(58) shows that ,in GCG case,when $t \gtrsim t_{\rm ds}$, expansion becomes de-Sitter like and and DE density acquires a constant value slightly less than present DE density. In non-GCG case, $\rho_{\rm de} \to 0$ as $t \to \infty$. From (48), it is found that $-0.63 > {\rm w}_{\rm de} \gtrsim -1$ taking β and ${\rm w}^0_{\rm de}$ given above. But for $t \geq t_{\rm ds}, {\rm w}_{\rm de} \simeq -1$.

Moreover, (53a) yields

$$\Big(\frac{\ddot{a}}{a}\Big)_{\rm GCG} = 0.73H_0^2|{\rm w}^0_{\rm de}|^{1/2(1+\beta)}\Big[1 - \frac{sech\theta}{\sqrt{1+2\beta}}\Big]^{-1}. \tag{60}$$

This equation shows that as t increases from $t = t_{*({\rm GCG})}$, $\Big(\frac{\ddot{a}}{a}\Big)_{\rm GCG}$ decreases and becomes $\simeq 0.73H_0^2|{\rm w}^0_{\rm de}|^{1/2(1+\beta)}$ for $t \geq t_{\rm ds}$. It means that, in GCG case, expansion is faster around transition time, but slows down as $t \to t_{\rm ds}$.

In non-GCG case, $a(t)$ is given by (36), which yields

$$\Big(\frac{\ddot{a}}{a}\Big)_{\rm non-GCG} = 0.73H_0^2\Big[1 - \frac{3(1+{\rm w}_{\rm de})}{2}\Big]\Big(\frac{a_0}{a}\Big)^{3(1+{\rm w}_{\rm de})}. \tag{61}$$

Thus, it is found that acceleration increases with growing scale factor $a(t)$ in both GCG and non-GCG cases. But, in GCG case, $\ddot{a} \propto a$ and $\ddot{a} \propto a^{1-3(1+{\rm w}_{\rm de})}$ for non-GCG case. It means that, in GCG case, acceleration grows faster with increasing $a(t)$ compared to the non-GCG case if ${\rm w}_{\rm de} > -1$. Its reason is explained in the following way.

Accelerated expansion is caused by DE, characterized by negative pressure violating the *strong energy condition*, which gives cosmic push due to its reverse gravity effect. In non-GCG case, $-p$ decreases with increasing scale factor (as $-p \propto \rho$), but it increases with $a(t)$ for GCG case and attains a constant value in finite time. So, GCG dark energy gives more cosmic push causing more acceleration compared to non-GCG dark energy,

4.In this letter, it is found dark energy emerges spontaneously from gravity sector using dual nature of the Ricci scalar as a geometrical field as well as physical field (mentioned above). Quintessence dark energy density, obtained here, falls from very high value $1.19 \times 10^{75}\mathrm{GeV}^4$ at the Planck scale to its current value $0.73\rho_{\mathrm{cr.}} \simeq 2.18 \times 10^{-47}\mathrm{GeV}^4$. At the transition time t_*, which is 8.59Gyr for $\mathrm{w_{de}} = -0.89$, dark energy begins to dominate giving a sudden jerk to the universe. As a result, a transition from deceleration to acceleration takes place showing the reverse gravity effect of dark energy. It is also found that dark radiation is washed away in the late universe. Transition from dominance of DM to dominance of DE is obtained at red-shift $0.47 < z_* < 0.599$ (for $-0.82 > \mathrm{w_{de}} > -1$) causing transition from deceleration to acceleration. Interestingly, z_*, obtained here, is consistent with the value of z_* given by 16 Type SNe Ia observations. Universe accelerates after $t > t_*$ with power-law expansion with vanishing energy density, when $t \to \infty$. But, if DE fluid behaves as GCG and barotropic fluid simultaneously, accelerated expansion is faster due to more cosmic push compared to non-GCG case. It is found that, in GCG case, expansion becomes de-Sitter like after a finite time with finite energy density.

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